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19. ABSTRACT (Continue on reverse if necessary and identify by block number) We have investigated four different approaches to the assessment of dimensionality of binary items. This has been done using a population model that allows sampling of both items and people and provides for variation and control of important parameters. We also consider the model to be realistic of performance of binary items used in tests of abilities.		
Indices of dimensionality based on the property of local independence of unidimensional tests distinguish between one factor and up to five factors with close to 100% accuracy in tests of 60 items administered to 2000 cases within the limits of variation of the parameters of distribution of item difficulties and of levels of factor intercorrelations. With the possible exception of full information factor analysis, our Local Independence Index is the most accurate available.		
Indices of dimensionality based on the pattern of second factor loadings derived from		
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simplex theory are also accurate under many combinations of parameters and are second in accuracy to Local Independence. These indices decrease in accuracy substantially at the highest level of factor correlations and the widest dispersion of item difficulties that we used.

The fitting of a simplex R-matrix to observed intercorrelations of binary items provides high accuracy under a wide range of parameters, but becomes highly sensitive to the combination of a high level of factor correlations and a wide distribution of item difficulties. An increase in sample size produces no increase in accuracy in the most unfavorable combination of parameters.

A quantitative index of the shape of the curve of successive Eigenvalues was used for matrices of phi coefficients, tetrachoric correlations, and variance-covariance matrices. None of these indices produced satisfactory accuracy except under most favorable combinations of parameters. Even so, the Eigenvalues of variance-covariance matrices provide a more accurate basis for a decision concerning dimensionality than tetrachoric correlations, which have been the statistics of choice. Tetrachorics are probably not dependable for any purpose when there is a wide range of item difficulties (or popularities) except in sample sizes substantially larger than 2,000.

Factor Analysis, Binary Items, 1961



1961	Q1	1961	Q1
DATA	DATA	DATA	DATA
Unreliability	Unreliability	Unreliability	Unreliability
Justification	Justification	Justification	Justification
2. Distribution			
3. Average Correlation			
4. Total Correlation			
Dist	Correlation	Dist	Correlation
A			

Introduction

In our research we have investigated indices of dimensionality of four distinct types and reported on indices representing three of these types in both our first technical report (Tucker, Humphreys, and Roznowski, 1986), and in the second (Roznowski, Humphreys, and Tucker, 1987).

Local Independence Indices

If a test is unidimensional, the items in the test are independent of each other in a sample of people all at the same level on the latent trait. This property of the test is local independence. Having the same raw score on the test can be used as an estimate of having the same score on the latent trait. Starting from this assumption two indices of dimensionality were tried. They are called Local Independence indices.

Pattern Indices

When the product-moment intercorrelations (phis) of a perfect Guttman scale are factored, the second factor loadings have a distinctive pattern of signs and relatives sizes. If the obtained intercorrelations are based on a single factor plus random error, the second factor loadings can be expected to approximate the expected pattern. In all, we tried four ways of obtaining a quantitative index of dimensionality from the pattern of second factor loadings. They are called Pattern indices.

Ratio of Differences Indices

Indices of dimensionality in the intercorrelations of continuous variates have long depended on relations among the successive Eigenvalues obtained in a principal factors analysis. Whether known as the "scree" test or "root staring," the assumption is that there will be a break in the curve of the Eigenvalues at the point the last replicable factor has been extracted. Thereafter the roots should have little slope. For the one factor case the

ratio of the difference between the first two roots to subsequent differences (we chose the average of the next two) should be large. We applied this principle to the roots of tetrachoric, product-moment, and variance-covariance matrices. These are the Ratio of Differences indices.

Root One Index

Even more popular than a break in the latent roots obtained in a principal factors analysis is the "root one" criterion in a principal components analysis. We did not look systematically at this criterion when applied to binary items for the very good reason that it was obviously inappropriate, given the low level of item intercorrelations in typical cognitive tests. One need only inspect a few samples to reject "root one" as a basis for a decision concerning dimensionality. The failure in binary data also indicates one important reason for failure in continuous data; that is, when replicable factors are determined by relatively small correlations, the criterion will fail.

Simplex Fitting Index

The items in a perfect Guttman scale form an R-matrix that has the perfect simplex form. It seemed possible that an index of dimensionality from the fit of the simplex model to the observed intercorrelations could be obtained. Although closely related in conception to the Pattern indices, we have placed it in a fourth category. Data concerning the Simplex Fitting index have not previously been made a matter of record.

The Population Model Used

An empirical investigation of criteria to distinguish between unidimensionality and multidimensionality as revealed in the intercorrelations of binary items requires a population model from which samples of both people and items can be drawn. Our model was described in considerable detail in our

first technical report (Tucker, Humphreys, and Roznowski, 1986). For present purposes it is sufficient to list the parameters of the model. In our research some of the parameters were varied; others were fixed.

The parameters varied were as follows: sample size, number of items, distribution of item difficulties, number of factors, and level of intercorrelations of the factors. We set other parameters so that the model reflected realistic psychological data. Some items were loaded on a single factor while others were complex. A sufficient number of factorially pure items was included to provide adequate factor definition for all numbers of factors. Provision was made for success by guessing. The size of item intercorrelations was set at levels typical of cognitive tests in which substantial unique variance is the norm. As number of factors, level of factor correlations, and distributions of item difficulties in the model were varied, variation in size of item correlations could not be held constant. Mean item correlations are presented in both published technical reports.

Factors in Continuous Variates

Insight into the nature of our model can be obtained by factoring intercorrelations of the variates prior to the point at which continuous variates are dichotomized to form binary ones. Table 1 presents Eigenvalues for three samples of 500 each for 30 variables for one through five principal factors in which squared multiple correlations served as communality estimates. For factors two through five intercorrelations were set, on average, at .55, which we considered to be at an intermediate oblique level. Also included in the table are estimated Eigenvalues for random data matrices based on the parallel analysis procedure of Montanelli and Humphreys (1976).

Table 1
 Successive Eigenvalues in One to Five Factors in the
 Continuous Model with Squared Multiple Correlations in the Diagonal
 (N=500)

Factors	Samples	Eigenvalues							
		1	2	3	4	5	6	7	8
1	1	19.42	.21	.15	.14	.14	.10	.10	.10
	2	17.68	.21	.18	.18	.12	.12	.10	.09
	3	19.32	.16	.14	.14	.12	.11	.09	.08
2	1	15.27	3.60	.17	.14	.13	.09	.06	.06
	2	15.08	3.55	.14	.14	.10	.09	.08	.07
	3	14.90	3.27	.18	.16	.13	.10	.09	.08
3	1	14.75	2.11	1.51	.17	.12	.11	.10	.08
	2	15.13	2.65	1.89	.15	.12	.11	.11	.08
	3	15.90	1.68	1.41	.16	.13	.12	.10	.09
4	1	14.05	1.91	1.46	.66	.15	.14	.10	.08
	2	15.95	1.24	1.08	.74	.17	.14	.11	.10
	3	15.65	1.08	.82	.78	.18	.17	.12	.10
5	1	13.81	1.50	1.14	.63	.50	.18	.14	.10
	2	15.22	1.22	.98	.79	.43	.12	.11	.09
	3	16.79	1.19	.74	.62	.43	.15	.13	.11
Parallel Analysis Estimates		.57	.48	.42	.38	.34	.32	.27	.24

The parameters for the tabled data were selected to represent intermediate levels of those used in the research reported in our second technical report. For the one factor case, however, we used the level of item intercorrelations on which the data in the first technical report were based. (The later change was made to bring the one factor correlations more nearly in line with the multiple factor correlations.)

For the continuous data of Table 1 there are breaks in the curve formed by the successive Eigenvalues for the proper number of factors in each sample. The parallel analysis criterion also leads to the expected number of factors. Eigenvalues for the same R-matrices in which unities have been placed in the diagonal, shown in Table 2, have similar breaks to those in Table 1 for the expected number of common factors, but the "root one" criterion fails after four common factors. Investigators who advocate a parallel analysis criterion in principal component analysis would commit additional errors. They would frequently accept only three factors in samples in which either four or five were required.

If the factors were more highly oblique, or if sample sizes were smaller, the number of factors decision would have been made with less confidence and probably less accuracy, for the continuous variates. Even with the parameters used the distribution of Eigenvalues looks increasingly unidimensional as the number of factors increases from two to five. It is clear that the number of factors decision would be made with more error in all combinations of parameters after the loss of information from the conversion of continuous variates to binary ones and from the introduction of success by guessing.

Table 2
 Successive Eigenvalues in One to Five Factors in the
 Continuous Model with Unities in the Diagonal
 (N=500)

		Eigenvalues							
Factors	Samples	1	2	3	4	5	6	7	8
1	1	19.76	.58	.54	.53	.52	.50	.48	.45
	2	18.07	.66	.66	.64	.58	.56	.53	.52
	3	19.66	.59	.57	.52	.51	.49	.47	.44
2	1	15.63	3.98	.61	.59	.58	.52	.49	.49
	2	15.45	3.93	.58	.56	.56	.52	.50	.49
	3	15.28	3.67	.62	.61	.60	.59	.54	.52
3	1	15.14	2.50	1.93	.59	.56	.53	.51	.51
	2	15.48	2.99	2.25	.55	.53	.52	.48	.46
	3	16.26	2.08	1.79	.58	.56	.55	.53	.49
4	1	14.46	2.31	1.87	1.10	.59	.58	.54	.52
	2	16.33	1.62	1.47	1.13	.58	.55	.54	.52
	3	16.04	1.48	1.25	1.19	.64	.60	.58	.55
5	1	14.24	1.94	1.61	1.08	.97	.66	.63	.56
	2	15.61	1.61	1.39	1.21	.89	.58	.53	.51
	3	17.15	1.55	1.12	.98	.80	.54	.53	.50

Evaluation of the Model

At this point one might ask the question whether our model is more complex than most real data matrices one is likely to encounter in practice. Our answer is negative. Investigators typically find clearcut simple structure in analyses of continuous measures only when they start with a good deal of information about their measures and select carefully a battery of tests on the basis of that information. Just as factorially pure tests are not in common supply, we do not expect to find commonly factorially pure items.

Intercorrelations among ability factors defined by continuous variates also tend to be high in wide ranges of talent. There are stable factor correlations in the ASVAB battery at the level of .55, for example, in which the factors are defined by tests having quite dissimilar content such as Arithmetic Reasoning and Mathematics Knowledge, on the one hand, and Vocabulary and Reading Comprehension, on the other. Not only is it unlikely that these four types of items would be found in a single item pool, but the two types that defined separate factors among total scores are unlikely to be found together. The selective factors imposed on item pools by the test constructor's conceptualization of the test are highly likely to produce high levels of obliqueness among multiple factors.

Finally, the emphasis in Item Response Theory on equivalent measurement accuracy at all levels of ability requires item pools having a wide range of item p-values. Without a wide range of difficulties in the pool item parameters are not well determined. Indices of dimensionality are expected to be applied to pools of binary items of which many are factorially complex, the multiple factors are highly correlated, and the items are widely different in levels of difficulty.

When cognitive data are viewed in terms of a hierarchical factor model, no serious error is committed by accepting a unidimensional hypothesis in the presence of multiple factors that are substantially oblique. The error becomes progressively less serious as the number of factors increases and as the factor correlations increase. Each of these parameters increases the contribution to variance of the general factor in the total score. Not only does the contribution to total variance of a group factor decrease as the number of factors increases, but the contribution to variance of the sum of five group factors is less than the contribution to variance of the sum of two group factors of the same degree of obliqueness. It seems counterintuitive, but as long as each item measures the general factor, the greater the factorial complexity of the test items the more closely does the total score reflect a single dimension. In many, many applications the most valid dimension is the one defined by the factor correlations. A test constructor should not choose to measure only one of the correlated factors in most applications.

Properly weighted multiple dimensions that are substantially positively correlated are not a problem for Item Response Theory when each examinee is exposed to every item as in a standard printed test. Multiple dimensions do become a problem, however, in adaptive testing. The expected build-up of the general or common variance in the total score only occurs when all secondary factors are adequately sampled. One cannot rely on an algorithm for the selection of items that depends only on the "a" and "b" parameters of the items in the pool. Secondary factors must be known or estimated and the information used in item selection in order to maximize the validity of the test as well as to avoid the bias that would result if all examinees were not measured on the same dominant dimension.

Organization of this Report

The remainder of this report contains three sections: a discussion of unpublished data that were reported at the 1987 ONR conference, an account of a generally unsuccessful attempt to compensate for the negative effects on indices of dimensionality of wide ranges of item difficulties, and a summary of what we have learned during the project, including our recommendation concerning a preferred index. The latter is supported in Appendix A by more complete data than we included in our second technical report.

Results Reported OrallyFitting a Simplex

We obtained data for 20 and 30 items on an index in which the simplex model was applied directly to the product-moment correlations of the binary items. These data were reported orally during the ONR conference at the University of South Carolina (1987). This index is identified as Simplex Fitting. The Pattern indices which appeared in our two technical reports are based on the characteristics of the principal factors extracted from a simplex R-matrix. We obtained least squares fits of the simplex model¹ to observed R-matrices for 100 samples each of all combinations of five factors, Ns of 125, 500, and 2000, three levels of distribution of item difficulties, and three levels of factor intercorrelations. For both of the last two parameters the levels were the same as those used in our second technical report.

The simplex model requires item product-moment correlations corrected for attenuation to have the property that all $r_{ik,j} = 0$, where $p_i > p_j > p_k$. By fixing only two reliabilities, those for the easiest and for the most difficult items, it is possible to obtain unique solutions for the remaining $n-2$ reliabilities and for $n-3$ true score correlations between items adjacent in difficulty. Given unique estimates of adjacent item true score

correlations, the remaining correlations in the estimated R-matrix are themselves uniquely determined. Not wishing the index to be confounded with the arbitrary fixing of the two reliabilities and, in consequence, two true score correlations, we decided at the outset to eliminate the $2(n-1)$ residuals associated with the most and least difficult items.

Results

Tables 3 and 4 contain statistics for 20 and 30 items, respectively, showing the degree of separation of the sampling distributions of one factor from factors two through five. This statistic is the same as the one described in our second technical report. A value of 200 represents no overlap between distributions while anything close to 100 represents essentially zero separation. It is seen that the accuracy with which one factor can be discriminated from multiple factors varies directly with sample size and number of items, and inversely with the number of multiple factors, degree of obliqueness of the factors, and the dispersion of item difficulties. These characteristics are all familiar; they are characteristics of every index we have tried although there is variation in sensitivity to levels of the five parameters from one index to another. Unfortunately, Simplex Fitting is more adversely affected by the dispersion of item difficulties than the others based on the simplex model. Given our conception of item pools in which an accurate decision concerning dimensionality is required, the accuracy of Simplex Fitting breaks down where it is needed most.

Table 3
 Discrimination of One versus Two through Five Factors by the Simplex Fitting Index
 as a Function of Factor Obliqueness, Item Difficulty Distribution, and Sample Size*

20 Items											
		Low			Intermediate			High			
		Nar	Med	Wide	Nar	Med	Wide	Nar	Med	Wide	
125	1 vs. 2	186	182	141	151	145	115	110	109	101	
	2 vs. 3	187	181	136	158	146	113	118	110	105	
	1 vs. 4	189	175	135	156	146	122	120	108	102	
	1 vs. 5	186	176	128	149	143	117	116	111	103	
500	1 vs. 2	200	195	162	198	185	126	187	172	110	
	1 vs. 3	200	197	173	200	189	123	186	167	103	
	1 vs. 4	200	196	164	200	185	117	185	163	106	
	1 vs. 5	200	194	154	195	182	115	178	164	106	
2000	1 vs. 2	200	200	182	200	198	151	200	194	129	
	1 vs. 3	200	199	181	200	199	148	200	192	126	
	1 vs. 4	200	199	177	200	197	148	200	191	115	
	1 vs. 5	200	199	170	200	196	139	200	193	113	

*From left to right in the table, the first column contains sample size, the second number of factors, and the third, fourth, and fifth distributions of item difficulties for low levels of obliqueness. Distributions of item difficulties are then repeated for intermediate and high obliqueness.

Table 4
 Discrimination of One versus Two through Five Factors by the Simplex Fitting Index
 as a Function of Factor Obliqueness, Item Difficulty Distribution, and Sample Size*

		30 Items									
		Low			Intermediate			High			
		Nar	Med	Wide	Nar	Med	Wide	Nar	Med	Wide	
125	1 vs. 2	188	188	147	165	154	123	122	113	103	
	1 vs. 3	189	189	149	163	152	127	123	110	101	
	1 vs. 4	188	188	139	158	138	122	113	104	105	
	1 vs. 5	182	182	135	157	141	117	110	110	101	
500	1 vs. 2	200	199	181	200	189	140	197	170	115	
	1 vs. 3	200	199	178	200	193	132	195	174	107	
	1 vs. 4	200	199	171	200	188	129	192	156	105	
	1 vs. 5	200	199	168	200	182	120	186	150	103	
2000	1 vs. 2	200	200	194	200	199	162	200	197	125	
	1 vs. 3	200	200	192	200	199	168	200	196	117	
	1 vs. 4	200	200	191	200	200	149	200	197	114	
	1 vs. 5	200	200	180	200	200	132	200	194	109	

*From left to right in the table, the first column contains sample size, the second number of factors, and the third, fourth, and fifth distributions of item difficulties for low levels of obliqueness. Distributions of item difficulties are then repeated for intermediate and high obliqueness.

Table 5 presents means and standard deviations of the sums of squared deviations of observed from expected for certain favorable combinations of parameters in the trimmed 20x20 and 30x30 matrices. Simplex Fitting can in many instances separate the sampling distributions of one factor and multiple factors very well indeed. The index is a good deal more variable in multiple factor R-matrices than in the unidimensional ones. It is also necessary to realize that the index is not normally distributed.

Table 6 includes parallel data for a selection of unfavorable combinations of parameters. The simplex model tends to fit correlations produced by highly oblique multiple factors among items varying widely in difficulty levels as well (or as poorly) as it fits R-matrices in which there is only one factor among widely distributed items.

A Possible Modification of the Methodology

We have considered whether the model fitting could be improved by fixing appropriately the true score correlations between items adjacent in difficulty. In a perfect Guttman scale the proportion passing the more difficult item is also the proportion passing both items. One could start with a model that fixed all such correlations at their maximum level for a unidimensional test. Although we have not tried this approach, it appears from inspection of the distributions of estimated adjacent item correlations in which only end reliabilities were fixed that improved accuracy would result; that is, the distributions of estimated adjacent item correlations have means and variances that also distinguish between one and multiple factors.

Table 5
 Means and Standard Deviations of the Simplex Fitting Index in
 Narrow Distributions of Item Difficulties and the Larger Samples

	Low		Intermediate		High	
	\bar{x}	Sx	\bar{x}	Sx	\bar{x}	Sx
20 Items						
1	.180	.025	.180	.025	.180	.025
2	1.080	.413	.556	.230	.312	.084
500	3	.913	.317	.478	.122	.289
	4	.814	.216	.421	.096	.280
	5	.773	.239	.420	.123	.268
30 Items						
1	.044	.007	.044	.007	.044	.007
2	.735	.312	.357	.151	.196	.070
2000	3	.669	.194	.363	.123	.168
	4	.606	.161	.276	.087	.149
	5	.561	.188	.281	.104	.141
20 Items						
1	.466	.035	.466	.035	.466	.035
2	2.532	.782	1.302	.393	.809	.207
500	3	2.266	.550	1.177	.294	.757
	4	1.742	.434	1.017	.206	.683
	5	1.659	.402	.953	.184	.632
30 Items						
1	.118	.011	.118	.011	.118	.011
2	1.803	.650	1.021	.319	.509	.146
2000	3	1.625	.434	.834	.254	.416
	4	1.347	.332	.665	.184	.343
	5	1.116	.263	.539	.135	.307

Table 6
Means and Standard Deviations of the Simplex Fitting Index in
Wide Distributions of Item Difficulties and the Larger Factor Correlations

		125		500		2000	
		\bar{x}	S_x	\bar{x}	S_x	\bar{x}	S_x
20 Items							
Inter- mediate	1	1.457	.591	.447	.205	.173	.109
	2	1.458	.470	.523	.196	.280	.101
	3	1.470	.495	.499	.222	.252	.085
	4	1.534	.577	.444	.140	.236	.079
	5	1.442	.339	.457	.187	.215	.086
High	1	1.457	.591	.447	.205	.173	.109
	2	1.302	.398	.388	.127	.184	.068
	3	1.344	.395	.396	.197	.161	.052
	4	1.407	.541	.393	.133	.152	.057
	5	1.410	.794	.383	.154	.154	.071
30 Items							
Inter- mediate	1	3.329	1.102	.978	.307	.392	.172
	2	3.708	1.730	1.236	.292	.681	.291
	3	3.363	.532	1.162	.277	.645	.154
	4	3.336	.708	1.055	.186	.510	.100
	5	3.148	.682	.990	.192	.462	.094
High	1	3.329	1.102	.978	.307	.392	.172
	2	3.165	.860	.977	.275	.442	.111
	3	3.100	.852	.886	.246	.391	.117
	4	3.040	.641	.868	.235	.360	.101
	5	2.894	.606	.847	.205	.318	.105

Unfortunately, it also looks as if the greatest improvement in accuracy would occur for combinations of parameters where the index is already highly accurate. In high obliqueness, wide distribution of item difficulties, and five factors, the adjacent item correlations estimated by the present methodology tend to converge on values being obtained for matrices based on a single factor. Perhaps, however, simplex fitting with adjacent item correlations fixed as described above should be tried systematically. If a maximum likelihood criterion of fitting were used, a chi square test of goodness of fit would be available. This test, however, when applied to samples from a model which is unidimensional except for measurement error, would vary as a function of dispersion of item difficulties. Guessing contributes more error variance in such data and is negatively correlated with total score.

Trying to Obtain Something for Nothing

Reduction in Spread of Item Difficulties

Because the adverse effects of wide distributions of item difficulty were so severe, we decided to try a method that would sacrifice power along certain dimensions for a possible gain from reducing the dispersion of item difficulties. We selected the higher two-thirds of the distribution of test scores and scored this sample on the more difficult two-thirds of the items, and repeated the procedure on the lower two-thirds of the scores and the easier two-thirds of the items. The overlapping one-third of the scores and items from the middle of the two distributions was designed to allow a test of the hypothesis that the two sets of items were measuring the same factor or factors. We sacrificed sample size and level of item intercorrelations (from restriction in range of talent) for a reduction in spread of item difficulties.

This procedure was tried out as described on a small scale for a highly unfavorable combination of parameters that included high oblique factors, a wide range of item difficulties, one, two, and five factors, 30 items, and an N of 500.

Thus two samples of approximately 333 cases each were scored on different but overlapping 20 item tests. We computed only two indices of dimensionality: Local Independence, our most accurate index in the research to date, and Simplex Fitting.²

Results

Table 7 contains a comparison of accuracy in both 20 and 30 items as heretofore computed with accuracy in each of the two 20 item tests selected by the procedure described from one 30 item test. One and five factors are represented by 100 replications, two factors by 50. In the case of Local Independence, there is a reduction in accuracy of discrimination for the modified index in comparison with its use in the standard manner in 30 items and 500 cases. Approximately equivalent accuracy is obtained when the comparison is with the standard computations for 20 items in 500 cases. Apparently the decrease in size of item intercorrelations in the narrower range of talent was a greater handicap than any gain associated with the decrease in dispersion of item difficulties.

The data for the Simplex Fitting index are not as easy to interpret. Programming for this task was divided between two persons who had no opportunity for face-to-face interaction. A few changes in the program were required. These changes produced a higher incidence of extreme outliers. A second difference, a failure to exclude residuals from the easiest and most difficult items, we were able to allow for in obtaining means and standard deviations in this table. In spite of the differences in outcomes for the two sets of data from the 30 Standard condition, it appears that the manipulation of items and samples produced a small increase in accuracy of diagnosis of dimensionality. We attribute this outcome to the high degree of sensitivity of this index to wide distributions of item difficulties. Perhaps a further gain in accuracy could be obtained from fixing the true score correlations as described earlier.

Table 7
 Comparison of Two Indices Computed in the Standard
 Manner with Their Counterparts after Selecting 20 Extreme Items in
 Each Direction from a 30 Item Test

	Factors					
	1		2		5	
	\bar{x}	S_x	\bar{x}	S_x	\bar{x}	S_x
Local Independence Index						
30 Standard	.705	.030	.455	.094	.680	.037
20/30 Difficult	.614	.053	.403	.105	.617	.053
20/30 Easy	.627	.058	.375	.121	.607	.057
20 Standard ^a	.697	.039	.545	.093	.658	.047
30 Standard ^a	.711	.028	.507	.089	.663	.042
Simplex Fitting Index ^b						
30 Standard ^c	.669	.473	1.238	1.467	1.073	1.640
20/30 Difficult ^c	.908	.314	1.618	.490	1.306	.506
20/30 Easy ^c	.996	.398	1.538	.532	1.268	.532
20 Standard ^a	.503	.212	.460	.153	.435	.160
30 Standard ^a	.467	.139	.467	.121	.408	.093

^a From earlier computer runs involving different samples of items and examinees.

^b The sum of squared deviations for 30 items was adjusted to the equivalent of a 20 item test.

^c The earlier computer program incorporated a method for avoiding most extreme outliers in the Simplex Fitting index. Note the evidence for extreme skewness for this index in the present computations.

What We Have Learned

The Recommended Index

The most accurate index available to an investigator, with the possible exception of full information factor analysis, is the one we have called Local Independence. Tables showing the accuracy of this index for all combinations of the parameters we varied appeared in our second technical report and are reprinted here as Tables 8-12. Although this index can be computed inexpensively, there is not a simple method of using it to make a decision concerning dimensionality in a set of binary items. We have no formula that allows estimation of means and standard deviations from combinations of parameters using settings that vary from the ones we used. We do believe that our selections do come close to defining the limits of the domain. To support use of the index we report in Appendix A the means and standard deviations for all combinations of parameters. In contrast, we included only a partial set in the second technical report.

Computation of the Local Independence Index

We also repeat here a step by step review of the methodology used in obtaining this index.

1. Compute separate variance-covariance matrices for each sample of persons who have the same total score on the test. Note that tetrachoric correlations are not involved.
2. Obtain an aggregate variance-covariance matrix by weighting each separate matrix by the size of its sample.
3. Change signs of the aggregate matrix by row and column until a maximum algebraic total of the matrix, excluding its principal diagonal, is obtained. This step is accomplished by a routine parallel in every way to the one used in centroid factor analysis.
4. The algebraic sum is now subtracted from the absolute sum of the residuals. This is the local Independence index.

Table 8

Discrimination of One Versus Two through Five Factors
 by the Local Independence Index as a Function of
 Factor Obliqueness, Item Difficulty, and Sample Size

		<u>20 Items</u>											
		LOW				INTERMEDIATE				HIGH			
		Narrow	Medium	Wide	Narrow	Medium	Wide	Narrow	Medium	Wide			
<u>125</u>	1 v. 2	188	186	174	173	169	145	149	132	121			
	1 v. 3	181	174	158	149	143	133	127	116	118			
	1 v. 4	163	162	148	141	135	133	126	113	110			
	1 v. 5	160	164	152	139	132	132	121	108	108			
<u>500</u>	1 v. 2	200	200	200	200	199	193	198	194	182			
	1 v. 3	199	198	197	191	195	183	190	183	159			
	1 v. 4	196	195	193	188	192	173	176	172	145			
	1 v. 5	193	197	183	175	181	155	166	154	136			
<u>2000</u>	1 v. 2	200	200	200	200	200	197	200	200	195			
	1 v. 3	200	200	200	200	200	196	200	197	191			
	1 v. 4	200	200	198	199	200	194	199	199	181			
	1 v. 5	200	200	196	199	200	189	199	195	180			

Note: 200 indicates total independence of distributions.

100 indicates total overlap.

Table 9

Discrimination of One Versus Two through Five Factors
by the Local Independence Index as a Function of
Factor Obliqueness, Item Difficulty, and Sample Size

		<u>30 Items</u>								
		LOW			INTERMEDIATE			HIGH		
		Narrow	Medium	Wide	Narrow	Medium	Wide	Narrow	Medium	Wide
<u>125</u>	1 v. 2	197	196	190	188	178	165	164	151	131
	1 v. 3	192	186	182	178	160	158	146	123	124
	1 v. 4	185	181	177	156	144	136	131	119	117
	1 v. 5	173	172	169	149	135	145	125	115	107
	<u>500</u>	200	200	200	200	200	200	199	200	197
<u>1000</u>	1 v. 2	200	200	200	196	200	195	195	188	177
	1 v. 3	200	200	200	194	195	188	187	195	157
	1 v. 4	198	199	199	192	190	176	170	171	154
	1 v. 5	195	199	197	199	200	199	200	200	192
<u>2000</u>	1 v. 2	200	200	200	200	200	200	200	200	200
	1 v. 3	200	200	200	200	200	200	200	200	196
	1 v. 4	200	200	199	199	200	199	200	200	192
	1 v. 5	200	200	200	200	200	196	199	200	189

Note: 200 indicates total independence of distributions.

100 indicates total overlap.

Table 10

Discrimination of One Versus Two through Five Factors
by the Local Independence Index as a Function of
Factor Obliqueness, Item Difficulty, and Sample Size

		<u>40 Items</u>											
		LOW				INTERMEDIATE				HIGH			
		Narrow	Medium	Wide	Narrow	Medium	Wide	Narrow	Medium	Wide			
<u>125</u>	1 v. 2	197	199	196	195	189	181	175	168	140			
	1 v. 3	194	191	187	184	176	165	147	148	123			
	1 v. 4	188	183	179	161	164	139	136	142	112			
	1 v. 5	177	187	169	156	154	123	128	125	111			
<u>500</u>	1 v. 2	200	200	200	200	200	199	200	199	200			
	1 v. 3	200	200	200	200	199	197	198	196	191			
	1 v. 4	200	200	200	200	196	196	194	192	167			
	1 v. 5	199	197	199	197	197	182	187	172	154			
<u>2000</u>	1 v. 2	200	200	200	200	200	200	200	200	200			
	1 v. 3	200	200	200	200	200	200	200	200	199			
	1 v. 4	200	200	200	200	200	199	200	200	198			
	1 v. 5	200	200	200	200	200	199	199	200	196			

Note: 200 indicates total independence of distributions.

100 indicates total overlap.

Table 11

Discrimination of One Versus Two through Five Factors
 by the Local Independence Index as a Function of
 Factor Obliqueness, Item Difficulty, and Sample Size

		<u>50 Items</u>								
		LOW			INTERMEDIATE			HIGH		
		Narrow	Medium	Wide	Narrow	Medium	Wide	Narrow	Medium	Wide
<u>125</u>	1 v. 2	200	200	198	196	191	174	176	169	149
	1 v. 3	200	197	187	187	184	162	162	149	131
	1 v. 4	197	191	183	174	168	138	152	133	113
	1 v. 5	187	184	177	170	151	134	141	124	106
<u>500</u>	1 v. 2	200	200	200	200	200	200	200	200	200
	1 v. 3	200	200	200	200	200	199	199	200	192
	1 v. 4	200	200	199	199	200	198	192	195	185
	1 v. 5	199	200	198	199	197	191	185	186	174
<u>2000</u>	1 v. 2	200	200	200	200	200	200	200	200	200
	1 v. 3	200	200	200	200	200	200	200	200	200
	1 v. 4	200	200	200	200	200	200	200	200	200
	1 v. 5	200	200	200	200	200	200	200	200	200

Note: 200 indicates total independence of distributions.

100 indicates total overlap.

Table 12

Discrimination of One Versus Two through Five Factors
 by the Local Independence Index as a Function of
 Factor Obliqueness, Item Difficulty, and Sample Size

		<u>60 Items</u>								
		LOW			INTERMEDIATE			HIGH		
		Narrow	Medium	Wide	Narrow	Medium	Wide	Narrow	Medium	Wide
<u>125</u>	1 v. 2	200	199	199	198	195	182	189	176	160
	1 v. 3	198	200	196	191	192	172	171	152	133
	1 v. 4	195	195	187	184	175	156	162	137	118
	1 v. 5	191	193	182	176	155	140	146	123	114
	<u>500</u>	1 v. 2	200	200	200	200	200	200	200	200
<u>500</u>	1 v. 3	200	200	200	200	200	200	200	200	198
	1 v. 4	200	200	200	200	200	199	199	199	187
	1 v. 5	199	200	200	200	199	197	192	190	176
	<u>2000</u>	1 v. 2	200	200	200	200	200	200	200	200
<u>2000</u>	1 v. 3	200	200	200	200	200	200	200	200	200
	1 v. 4	200	200	200	200	200	200	200	200	200
	1 v. 5	200	200	200	200	200	200	200	200	199

Note: 200 indicates total independence of distributions.

100 indicates total overlap.

If the difference between the absolute and algebraic sums is small, there is order in the aggregate matrix arising from more than one factor in the covariances; that is, the items of persons having the same levels of estimated ability are not locally independent. Given local independence, however, there is no order in that matrix, and the difference between the absolute and algebraic sums is large.

Properties of the Index

It was not surprising that Local Independence followed the pattern expected of sample statistics and became more accurate with increases in sample size. The number of items in the pool was also an important parameter, but its contribution to accuracy is presumably by way of the number of items per factor. The latter is a possible parameter that was not varied independently of number of items. A test constructor starts with a conception of a test and writes multiple items. If the conception produces multiple factors, the difference in number of factors in pools of 20 and 60 items based on the same conception should be trivial.

It was also not surprising that the number of factors in the model was negatively related to the degree of separation of the sampling distributions of one and multiple factors. As described earlier, when multiple factors are substantially oblique the total score becomes increasingly a more accurate measure of the second-order factor defined by the multiple first-order factors.

The parameters of distribution of item difficulties and of factor intercorrelations are almost impossible to handle in any generalizable way. We arbitrarily defined three levels of each. These levels are briefly described here as well as in our second technical report. Factor intercorrelations: low .35, intermediate .55, and high .70, each on average. Item difficulty distribution means and standard deviations in that order, respectively, in normal deviate units: narrow -.13 and .32, medium .00 and .50, and wide .10 and .80. Item difficulty distribution means and standard deviations in proportion correct following the

effects of guessing: narrow .615 and .113, medium .580 and .155, and wide .567 and .215.

Local Independence shows less separation of one from multiple factors as both factor obliqueness and dispersion of item difficulties increase, but it is a well-behaved index, lacking complex interactions among the five parameters. The combination of 60 items in a sample of 2,000 allowed this index to achieve virtually complete separation of one from multiple factors under the most unfavorable combination of the other parameters.

Indices Dependent on Tetrachoric Correlations

We have also learned that relationships among successive Eigenvalues of R-matrices composed of tetrachoric correlations having either unities or estimated communalities down the principal diagonal are a highly undependable guide to the dimensionality of binary items. The information about dimensionality obtained from the Eigenvalues of tetrachorics is only a little more accurate than that obtained from product-moment correlations (phis) and is less accurate, within the limits of the values of the parameters we used, than the application of the same index to the Eigenvalues of the variance-covariance matrix. Data to support these assertions were reported in technical reports 1 and 2 and are not repeated here.

Of course, indices based on tetrachorics do improve in accuracy as sample size increases. It is possible that the accuracy of indices involving tetrachorics might become competitive with others if sample size were increased substantially above our maximum value of 2,000, but this can hardly be recommended to test constructors. The heart of the problem is the sampling error of a tetrachoric correlation, which is a function of sample size, size of the population correlation, and the two item difficulties. If, for example, the correlation is between an easy item ($p = .9$) and a difficult item ($p = .1$), a shift of one case in a hundred changes a correlation of +1.0 to .0.

More generally, we have not found any way of manipulating Eigenvalues that is competitive in accuracy to indices in other categories over the full range of combinations of our parameters. The best that can be said for indices in this category is that they are not as sensitive to decreases in sample size and number of items as the rest.

Indices Based on Simplex Theory

Both the Pattern Indices and Simplex Fitting are intermediate in accuracy overall to Local Independence and Ratio of Differences Indices. The former two, however, performed in a disappointing fashion under combinations of high levels of factor intercorrelations and a wide dispersion of item difficulties. There also seems to be an interaction of these parameters with sample size in the direction of little or no gain in accuracy with increase in N in unfavorable combinations. Early in our research, when we were using only the low and intermediate levels of factor intercorrelations, the Pattern indices were found to be less sensitive to dispersion of item difficulties than Local Independence. Extrapolation to larger N s and more items than we had used up to that point in time suggested more promise in these approaches than was eventually realized. Apparently the second-order factor becomes more and more evident in the correlations under the conditions described. The Local Independence index, however, is able to find order in the multiple factor situation in spite of the simplex appearance of the R-matrix.

Footnotes

¹ The authors wish to express their thanks to Dr. Timothy Davey for his help in programming the Simplex Fitting index.

² Mr. Gary Thomasson made a major contribution to the completion of this task.

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Table A-1

Means and Standard Deviations of the Local Independence Index
for All Combinations of Parameters*

20 Items

		Low	Med	Wide	Intermediate			High	Med	Wide
					Nar	Med	Wide			
<u>125</u>	1	580	579	554	580	579	554	580	579	554
		055	053	064	055	053	064	055	053	064
	2	301	338	357	422	433	475	501	527	533
		118	117	121	105	113	092	081	087	077
	3	377	420	432	484	509	500	543	561	543
		112	099	095	106	082	079	073	064	069
	4	450	459	460	533	531	511	554	564	550
		099	091	094	062	069	074	060	064	061
	5	479	460	459	532	540	509	565	576	555
		085	095	090	069	059	076	047	061	055
<u>500</u>	1	699	707	697	699	707	697	699	707	697
		039	034	039	039	034	039	039	034	039
	2	189	260	331	298	370	447	434	472	545
		077	085	100	105	099	116	095	100	093
	3	383	388	454	450	486	535	535	578	611
		091	102	082	104	093	088	077	076	063
	4	478	497	519	535	546	586	603	613	642
		077	080	072	081	071	067	068	067	062
	5	541	541	567	587	600	632	625	648	658
		057	060	065	060	054	052	050	047	047
<u>2000</u>	1	818	819	805	818	819	805	818	819	805
		021	021	032	021	021	032	021	021	032
	2	169	218	298	274	319	436	376	443	545
		077	076	105	101	115	111	106	107	096
	3	374	415	456	398	471	557	523	589	643
		110	102	095	098	088	086	105	090	072
	4	471	501	532	527	575	608	621	649	703
		085	084	078	081	074	093	073	060	063
	5	542	574	606	599	627	674	665	694	725
		067	064	060	054	059	047	047	042	045

* From left to right in the table, the first column contains sample size, the second number of factors, and the third, fourth, and fifth distributions of item difficulties for low levels of obliqueness. Distributions of item difficulties are then repeated for intermediate and high obliqueness. The first row opposite the factor number contains means, the second the standard deviations. Both are reported to three significant decimal places, but decimal points are omitted to reduce crowding.

Table A-2

Means and Standard Deviations of the Local Independence Index
for All Combinations of Parameters*

30 Items

		Nar	Low			Intermediate			High		
			Med	Wide	Nar	Med	Wide	Nar	Med	Wide	
<u>125</u>	1	628	623	602	628	623	602	628	623	602	
		040	045	043	040	045	043	040	045	043	
	2	302	320	379	417	451	487	516	542	567	
		103	105	098	101	098	092	088	080	070	
	3	414	426	442	496	525	518	572	599	582	
		094	088	086	078	079	081	065	055	056	
	4	482	482	473	548	562	565	596	604	591	
		073	072	077	066	070	060	061	044	045	
	5	510	510	493	573	583	558	606	609	603	
		068	067	077	058	057	054	042	042	045	
<u>500</u>	1	706	710	711	706	710	711	706	710	711	
		028	028	028	028	028	028	028	028	028	
	2	156	193	270	263	303	389	375	438	507	
		062	079	091	085	093	083	101	083	089	
	3	364	388	409	447	454	510	521	548	603	
		095	090	089	089	081	079	083	064	063	
	4	464	463	490	530	546	568	580	617	652	
		075	079	075	069	065	077	061	067	052	
	5	503	529	541	559	590	618	628	642	663	
		071	065	067	064	053	057	050	044	042	
<u>2000</u>	1	803	808	797	803	808	797	803	808	797	
		018	017	025	018	017	025	018	017	025	
	2	113	135	208	164	217	332	270	333	440	
		055	067	077	061	079	108	083	088	091	
	3	348	367	430	386	427	472	474	506	593	
		116	099	101	086	104	086	089	075	079	
	4	450	451	514	481	503	572	550	584	655	
		091	081	084	077	070	079	076	059	059	
	5	497	524	563	550	582	631	608	650	696	
		065	072	072	065	067	056	061	051	045	

* From left to right in the table, the first column contains sample size, the second number of factors, and the third, fourth, and fifth distributions of item difficulties for low levels of obliqueness. Distributions of item difficulties are then repeated for intermediate and high obliqueness. The first row opposite the factor number contains means, the second the standard deviations. Both are reported to three significant decimal places, but decimal points are omitted to reduce crowding.

Table A-3

Means and Standard Deviations of the Local Independence Index
for All Combinations of Parameters*

40 Items

		Low			Intermediate			High		
		Nar	Med	Wide	Nar	Med	Wide	Nar	Med	Wide
<u>125</u>	1	655	654	628	655	654	628	655	654	628
		037	033	040	037	033	040	037	033	040
	2	327	348	375	426	469	485	533	555	583
		096	098	092	099	099	083	072	072	062
	3	428	459	464	511	546	539	594	608	605
		088	096	078	076	069	062	058	052	048
	4	488	505	504	575	582	583	616	626	620
		072	085	074	053	058	049	050	037	044
	5	546	534	532	592	602	600	628	633	623
		056	059	063	049	047	051	045	037	035
<u>500</u>	1	723	724	718	723	724	718	723	724	718
		023	024	024	023	024	024	023	024	024
	2	150	171	256	230	291	369	340	401	477
		056	061	081	065	087	089	079	094	074
	3	371	378	419	424	463	496	501	541	585
		098	091	087	087	082	080	075	077	067
	4	470	458	505	504	536	574	581	607	647
		074	069	062	072	071	055	067	050	047
	5	516	533	548	559	586	614	631	652	670
		062	063	058	058	055	051	050	051	036
<u>2000</u>	1	797	802	797	797	802	797	797	802	797
		016	015	022	016	015	022	016	015	022
	2	083	101	162	133	161	256	226	282	372
		035	042	056	052	060	086	072	081	088
	3	357	360	383	385	386	461	422	475	540
		116	100	084	100	099	093	078	083	074
	4	434	440	466	470	497	552	539	563	616
		088	081	072	077	078	068	061	060	055
	5	490	507	543	534	548	599	598	627	667
		073	079	068	067	065	068	065	047	053

* From left to right in the table, the first column contains sample size, the second number of factors, and the third, fourth, and fifth distributions of item difficulties for low levels of obliqueness. Distributions of item difficulties are then repeated for intermediate and high obliqueness. The first row opposite the factor number contains means, the second the standard deviations. Both are reported to three significant decimal places, but decimal points are omitted to reduce crowding.

Table A-4

Means and Standard Deviations of the Local Independence Index
for All Combinations of Parameters*

50 Items

			Low			Intermediate			High		
			Nar	Med	Wide	Nar	Med	Wide	Nar	Med	Wide
						681	672	646	681	672	646
<u>125</u>	1	681	672	646		681	672	646	681	672	646
		028	029	036		028	029	036	028	029	036
	2	311	333	395		437	475	509	554	576	588
		099	085	090		097	097	092	080	066	060
	3	441	453	483		532	553	558	606	628	616
		074	087	079		071	065	065	067	046	044
	4	508	526	525		590	597	603	631	644	635
		064	068	063		057	055	051	044	039	039
	5	542	553	538		615	621	610	650	654	643
		067	064	058		048	049	047	037	036	038
<u>500</u>	1	734	737	733		734	737	733	734	737	733
		023	022	020		023	022	020	023	022	020
	2	048	164	233		230	277	345	353	385	470
		247	047	061		070	081	085	076	066	072
	3	380	389	431		429	450	507	514	539	592
		097	081	081		076	075	077	073	061	061
	4	458	470	492		513	540	575	598	617	647
		072	074	068		070	062	053	059	048	044
	5	513	528	553		577	591	626	636	649	681
		064	061	064		053	052	051	054	041	034
<u>2000</u>	1	798	801	795		798	801	795	798	801	795
		014	015	017		014	015	017	014	015	017
	2	067	089	139		122	142	219	195	236	322
		030	032	048		048	050	069	058	065	080
	3	359	363	390		371	409	447	427	458	520
		090	091	103		102	098	089	091	077	070
	4	439	434	473		473	479	532	534	543	602
		088	078	076		078	064	068	068	062	060
	5	494	497	522		528	551	576	584	617	657
		068	066	060		061	062	059	050	049	045

* From left to right in the table, the first column contains sample size, the second number of factors, and the third, fourth, and fifth distributions of item difficulties for low levels of obliqueness. Distributions of item difficulties are then repeated for intermediate and high obliqueness. The first row opposite the factor number contains means, the second the standard deviations. Both are reported to three significant decimal places, but decimal points are omitted to reduce crowding.

Table A-5

Means and Standard Deviations of the Local Independence Index
for All Combinations of Parameters*

60 Items

		Low	Med	Wide	Intermediate			High		
					Nar	Med	Wide	Nar	Med	Wide
<u>125</u>	1	693	690	664	693	690	664	693	690	664
		024	024	030	024	024	030	024	024	030
	2	326	362	377	449	488	517	555	588	594
		086	093	103	084	091	083	075	069	062
	3	467	472	467	552	561	577	619	637	631
		075	073	073	072	058	059	057	048	051
	4	522	523	525	596	612	606	645	661	653
		070	067	064	053	052	050	042	036	037
	5	565	569	551	622	638	632	666	672	655
		062	049	057	047	043	036	028	030	033
<u>500</u>	1	753	754	747	753	754	747	753	754	747
		018	016	020	018	016	020	018	016	020
	2	135	168	225	223	253	351	328	382	459
		046	055	065	055	069	073	074	086	074
	3	369	393	429	436	454	501	504	542	594
		094	080	073	087	074	071	067	066	065
	4	468	483	490	509	550	589	596	613	656
		072	059	063	053	053	053	061	049	042
	5	532	538	575	583	596	627	642	664	689
		061	055	056	048	052	055	048	043	032
<u>2000</u>	1	801	803	797	801	803	797	801	803	797
		014	014	014	014	014	014	014	014	014
	2	062	086	126	117	137	206	181	223	321
		023	034	037	039	043	073	056	069	076
	3	367	361	379	371	410	442	439	444	520
		098	101	103	098	087	089	076	085	075
	4	447	453	488	470	476	521	529	542	604
		076	069	071	074	068	062	065	064	052
	5	506	501	537	506	544	572	589	608	648
		071	060	058	059	060	055	053	053	048

* From left to right in the table, the first column contains sample size, the second number of factors, and the third, fourth, and fifth distributions of item difficulties for low levels of obliqueness. Distributions of item difficulties are then repeated for intermediate and high obliqueness. The first row opposite the factor number contains means, the second the standard deviations. Both are reported to three significant decimal places, but decimal points are omitted to reduce crowding.

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